How to modify psychopathological states? Hypotheses based on complex systems theory

Hermann Haken* and Wolfgang Tschacher**

*Institute for Theoretical Physics, University of Stuttgart, Germany
cos@itp1.uni-stuttgart.de
**University Hospital of Psychiatry and Psychotherapy, University of Bern, Switzerland
wolfgang.tschacher@spk.unibe.ch

corresponding author: Wolfgang Tschacher

pre-publication version

Abstract

In our mathematical analysis based on the assumptions of complexity science, the emergence of a pattern is the result of a competition of modes, which each have a parameter value attached. In the context of visual pattern recognition, a specific connectionist system (the synergetic computer SC) was developed, which was derived from the assumptions of synergetics, a theory of complex systems. We adapted the processes of visual pattern recognition performed by the SC to a different context, psychopathology and therapeutic interventions, assuming these scenarios are analogous. The problem then becomes, under which conditions will a previously established psychopathological pattern not be restituted? We discuss several cases by using the equations of the SC. Translated to the psychopathological context, we interpret the mathematical findings and proofs in such a way that successful corrective interventions, e.g. by psychotherapy, should focus on one alternative pattern only. This alternative cognition-behavior-experience pattern is to be constructed individually by a therapist and a patient in the therapeutic alliance. The alternative pattern must be provided with higher valence (i.e. affective and motivational intensity) than possessed by the psychopathological pattern. Our findings do not support a linear symptom-oriented therapy approach based on specific intervention techniques, but rather a holistic approach. This is consistent with empirical results of psychotherapy research, especially the theory of common factors.

Keywords: psychotherapy, psychopathological disorder, synergetics, self-organization, affordance, common factors

Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 21, No. 1, pp. 19-34.

Introduction

Enduring psychopathological problems, such as personality disorders, generalized or phobic anxiety, or obsessive-compulsive disorder, are commonly defined as syndromes, i.e. lists of attributes or symptoms. According to a dimensional view in psychopathology, each attribute can in principle be quantified and measured using one or several variables. Such variables may come in the shape of psychometric ratings, of physiological or behavioral measures. Thus, any psychopathological syndrome can be represented by a vector $\vec{v} = (v_1, ..., v_L)$, which comprises the totality of variables v that make up the syndrome. In general, the goal of psychotherapeutic and psychiatric treatment is to generate stable states in a patient that are different from this syndromatic state and that are associated with less suffering and higher quality of life.

The synergetic computer (SC) is an algorithm that was originally developed to study processes of pattern recognition in visual perception (Haken, 2004). The SC is a self-organizing artificial neural network (Kohonen, 1987) that can be implemented on a digital computer in the same way as neurocomputers or neural nets (including the recently developed deep learning architectures) are actually algorithms running on digital computers. The SC has three layers, an input, an output and a hidden layer. Other than in feed-forward Hopfield nets, the nodes of the hidden layer of the SC are coupled. The SC was specifically designed to yield nonambiguous responses and not get stuck in local minima. This premise of network design was chosen to approximate a realistic model of cognitive and neuronal functioning, assuming that real cognition relies on fast unequivocal decisions. Connectionist remedies for avoiding local minima, such as simulated annealing, are presumably not what happens in real cognition (nor in the brain). Nearly all neural network approaches are plagued by occurrences of such 'ghost states'. The SC is free of such states because of its construction that guarantees a one-to-one correspondence between the fixed point attractors and the learned/stored patterns. Mathematically, the SC algorithm consists of a set of coupled nonlinear equations that describe the temporal evolution of the activities of the components of a complex system, e.g. the activities of neurons.

The attribute 'synergetic' is owed to the fact that the formulation of the SC equations is inspired by equations that appear in models of fluid dynamics or biological morphogenesis and are dealt with, from a unifying point of view, by a field called synergetics (Haken, 1977). Synergetics specifically addresses those processes that give rise to the formation of macroscopic spatial or temporal patterns. Synergetics provides a mathematical framework describing self-organization dynamics in non-equilibrium environments.The SC formulation capitalizes on a close analogy between pattern formation (morphogenesis) and pattern recognition. In both cases, the detailed and complicated dynamics of system components (e.g. particles of a fluid or neurons in a brain) can be reduced to a set of much fewer variables, which govern the collective behavior, and are called order parameters. Their competition is of a 'winner take all' type that decides which pattern is formed, or in the case of pattern recognition, recognized. The outcome of this competion is determined by initial conditions and typical system parameters.

Based on these principles, the SC was implemented as a device to reconstruct a prototypical, learned pattern (e.g., a face) on the basis of incomplete or distorted information (Ditzinger & Haken, 1989). With respect to this completion dynamics, the SC is a synergetic model of the functioning of the visual brain that can 'recognize' familiar faces (the previously learned prototypes) even when the available information is low or degraded, e.g. when there is little light or when a presented face is partially occluded.

In biology, systems frequently apply the same strategies to solve different, but related, problems. Our general assumption is that in psychopathology we may encounter a system that functions analogous to the recognition of visual patterns by an associative network in perception. In psychopathology, a specific pattern of behaviors, cognitions, and experiences represents the learned and stored prototype, a fixed pattern of the full manifestation of a psychopathological disorder. The patient's problem is accordingly that under many if not all circumstances this pattern will be established again and again. The goal of therapy therefore is the inverse of the goal of visual recognition; the therapeutic goal is to find conditions under which the system no longer 'recognizes the prototype', i.e. will *not* reconstruct the psychopathological pattern. In the following, we wish to build on these analogies between visual patterns and patterns of symptoms.

As an example of a psychopathological disorder, let us use "Borderline personality disorder" (BPD), a condition that has attracted a large volume of research and for which several specialized treatment routines have been developed. According to the World Health Organization's classification of diseases, BPD is present when at least three of five 'impulsive personality' criteria are fulfilled, and, in addition, at least two of six specific borderline criteria. We label these criteria $q_1,...,q_{11}$ to prepare our argumentation in the following (Table 1).

Table 1: The WHO criteria of the psychopathological pattern "Borderline personality disorder"

Impulsive personality criteria:

q_1 : marked tendency to act unexpectedly and without consideration of the consequences;

q_2 : marked tendency to engage in quarrelsome behavior

q_3 : liability to outbursts of anger or violence

q_4 : difficulty in maintaining any course of action that offers no immediate reward

q_5 : unstable and capricious (impulsive, whimsical) mood.

Specific BPD criteria:

q_6 : disturbances in and uncertainty about self-image

q_7 : liability to become involved in intense and unstable relationships

q_8 : excessive efforts to avoid abandonment

q_9 : recurrent threats or acts of self-harm

q_{10} : chronic feelings of emptiness

q_{11} : demonstrates impulsive behavior, e.g., speeding, substance abuse.

In clinical settings, especially in the context of classifications in psychiatry, symptoms are often used as if they were categories with only two truth values attached (fulfilled / not fulfilled). This is mirrored in the official terminology of the criteria given above. However, multiple shortcomings of such a conceptualization have been discussed: First, the criteria are rather vague and subject to interpretation. Second, the resulting diagnosis is categorical instead of dimensional, so that of two persons experiencing almost the same severity of symptoms one may be considered to have the full disorder and the other no disorder at all. Third, a selection of 5 out of 11 criteria leaves room for hundreds (exactly, 462) of different patterns that are all supposed to nevertheless denote a single disorder, BPD.

Yet it is not our intention to generally discuss the pros and cons of categorical psychiatric diagnoses here. Let us nevertheless assume for the present purpose that it is possible to transform the eleven categories to quantitative scales $q_1,...,q_{11}$ that operationalize each of the eleven attributes. All attributes of BPD together can thus be represented by a vector $\vec{q}$ with eleven components. Each component $q_1,...,q_{11}$ denotes the cognitive, behavioral, or experiential-emotional state that an individual may occupy with respect to the attributes of BPD, and the totality of these states is expressed by the state vector $\vec{q}$. Each vector is a cognition-behavior-experience state of an individual.

In the SC, different vectors $\vec{v}_k$ are stored in associative memory after learning trials with prototypical stimulus patterns or, in our present application, prototypical symptom patterns. Each $\vec{v}_k$ has a corresponding parameter λ_k that was interpreted as an 'attention parameter' in previous research on the perception of bistable visual images (Ditzinger & Haken, 1989) and scenes (Fuchs & Haken, 1988). An attention parameter is a purely mathematical construct of synergetics; in face recognition tasks it may denote the amount of attentional cognitive resources that are connected to a given face $\vec{v}_k$. If the attention for one of the possible solutions to an ambiguous stimulus pattern is raised (e.g., by some priming input), the probability that this particular pattern arises is increased. Hence, in psychological terms, λ_k is a motivational parameter that we relate to the valence or affordance (Haken & Tschacher, 2011) a certain face has for the perceiver. In the following we will use the term 'valence' for this typical system parameter of the SC, underlining the motivational and affective character of this term in the context of psychological dynamics. Valence (originally *Aufforderungscharakter*) was introduced by Kurt Lewin (1936) in his topological psychology to denote a force in the life space of an individual. This concept was later adopted by Lewin's disciple J.J. Gibson (1979), who translated it as affordance. Roughly speaking, in ecological terms, the system dynamics originates from the competition for information resources where efficiency is controlled by the valence parameters. In the context of psychopathology, $\vec{v}_k$ denotes an established psychopathological pattern such as BPD that is 'stored in the memory' of an individual because of his or her learning history – one may think of $\vec{v}_k$ as an attractor in the eleven-dimensional phase space of BPD. The valence parameters λ_k represent the attraction or salience of this pattern for the individual person.

State space dynamics is the key to the ideas developed in the following. The SC equations are of the type of evolution equations, like the well-known Lotka-Volterra equations in population dynamics (Choi, 1997). Yet, whereas the Lotka-Volterra equations allow for oscillatory solutions (predator-prey dynamics) as well as for extinction, the SC equations are constructed in such a way that they only permit an (oscillation-free) gradient dynamics leading to fixed point attractors. The SC equations may however be generalized so that the attention/valence parameters are subjected to an additional dynamics, and oscillations can occur (Ditzinger & Haken, 1989). The possibility of chaotic dynamics has not been studied yet.

The SC state space dynamics starts with a certain constellation given by an initial state vector $\vec{q}(0)$ constituted by attributes $q_1,...,q_L$. $\vec{q}(0)$ describes any initial state of an individual agent and is given by the initial instantiations in all single attributes. This initial state, which may be an entirely non-symptomatic, healthy state of an individual, then induces a dynamics of responses of the agent, which we can model using the SC. Our mathematical analysis of this dynamics is the main topic of this article. Based on the previous work with visual pattern recognition using the SC, we developed the following intuitions for the psychopathological applications.

These intuitions may be formulated as hypotheses that can be tested (using mathematical proof) in the framework of the SC: If only one valence parameter $\lambda_1 > 0$ exists, and $\vec{v}_1$ is the psychopathological pattern, any arbitrarily small overlap between $\vec{v}_1$ and an initial state $\vec{q}(0)$ suffices for this pattern to develop completely. In other words, any minimal incident is sufficient for the development of the full prototype disorder. Our second hypothesis says that even in the presence of further $\lambda_k > 0$, many initial situations will again result in a winning $\vec{v}_1$. From the analysis of these hypotheses we will derive prerequisites for how the restitution of the disorder can be prevented by therapy.

Methods and mathematical treatment

The mathematical apparatus of the model is the algorithm of the synergetic computer (SC) as described in Haken (2004). *M* is the number of prototype patterns $\vec{v}_k$ stored in the SC. Prototype patterns were established through previous learning trials, with $k = 1,...,M$. $\vec{v}_k^+$ are the adjoint vectors of the prototype patterns. λ_k are the attention parameters in the context of visual recognition, in our present terminology λ_k are the 'valence parameters' or 'valences' for the development of a disorder in the context of psychopathological patterns.

The following equation (1) is an *ansatz* formulated in analogy to self-organized pattern formation in fluid dynamics. (1) describes how a system state $\vec{q}$ changes in time. *B* and *C* are positive constants. The brackets (..) contain scalar products of vectors that result in numbers. This equation will not be explained in detail here (but see Ditzinger & Haken, 1989); it may suffice to say that the initial term in (1) is a projection of a state onto a prototype pattern depending on the respective valences, the second term is a discrimination function, and the third term is a saturation function that keeps the system dynamics within reasonable bounds.

$$\frac{d\vec{q}}{dt} = \sum_{k=1}^{M} \lambda_k \vec{v}_k (\vec{v}_k^+ \vec{q}) - B \sum_{k' \neq k}^{M} (\vec{v}_{k'}^+, \vec{q})^2 (\vec{q}^+ \vec{v}_k) \vec{v}_k - C(\vec{q}^+ \vec{q})\vec{q} \qquad (1)$$

The differential equation (1) can be transformed into a combination of more or less stable modes. Synergetics rests on the finding that the unstable modes vanish with time, which leaves the stable modes called 'order parameters' ξ_k (Haken, 1977). We again will not go into the details here, but order parameters are dynamical variables that are assigned to patterns. To use the Necker cube of gestalt psychology as an example (Fig. 1), the visual stimulus of the cube (left image in Fig. 1) gives rise to two order parameters (right images in Fig. 1) that exemplify the two possible modes of viewing the cube in three dimensions.

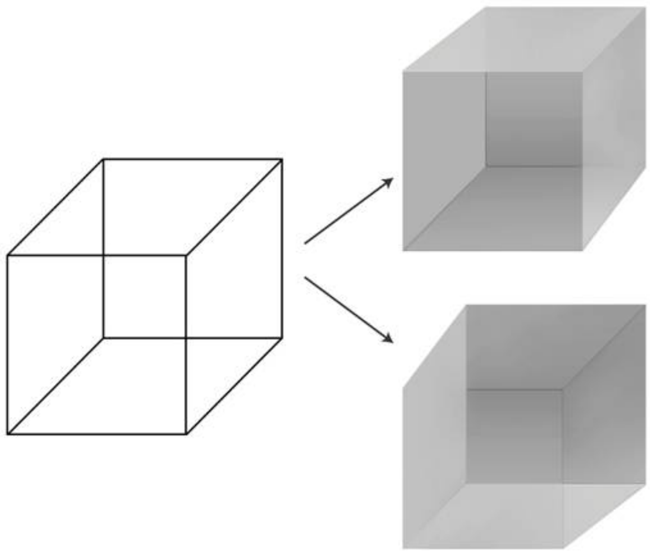

Figure 1. The Necker cube (left) may be viewed in two different ways or modes (right). These two modes are the competing order parameters of the visual system when the two-dimensional stimulus (left) is presented.

Due to synergetics, the unstable modes die out, so that it suffices to consider the order parameters alone, which are defined by

$$\xi_k = (\vec{v}_k^{+}\vec{q}), \ \lambda_k > 0 \tag{2}$$

and the state vector of an individual at any point in time therefore becomes

$$\vec{q}(t) = \sum_{k=1}^{M} \xi_k(t)\vec{v}_k \tag{3}$$

The system dynamics of (1) is thus simplified with the use of (2), (3) and reads

$$\frac{d\xi_k}{dt} = (\lambda_k - B\sum_{k' \neq k}^{M} \xi_{k'}^2 - C\sum_{k'}^{M} \xi_{k'}^2)\xi_k \tag{4}$$

Equation (4), the order parameter equation, describes that the change of order parameters depends on the valences, on the competition between patterns $B\sum_{k' \neq k}^{M} \xi_{k'}^2$, and on the saturation term $C\sum_{k'}^{M} \xi_{k'}^2$. In other words, system evolution expressed by the order parameters is a mixture of excitatory and damping (inhibitory) influences. The guiding question can now be formulated in a simpler fashion – how does this mixture depend on the valences λ_k?

Case 1: One psychopathological attractor

We are dealing with only one pattern of psychopathology that was pre-established in the system's memory, i.e. M=1 and $\xi \equiv \xi_1$ (index "1" dropped in (4)). The system dynamics thus is

$$\frac{d\xi}{dt} = \lambda\xi - C\xi^3 \qquad (5)$$

Equation (5) is a normal form that is often used in synergetics (Haken, 1977) and catastrophe theory (Guastello, 1995) to express attractor dynamics.

$\vec{q}(0)$ is the initial state of a person with respect to the vector of attributes of psychopathology. Then $\xi_0 \equiv \xi(0) = (\vec{v}^+\vec{q}(0))$. Even if ξ_0 is very small but $\neq 0$, the exact solution to the differential equation (5) reads

$$\xi(t) = \sqrt{\lambda/C}\,\xi_0(\xi_0^2 + (\lambda/C - \xi_0^2)e^{-\lambda t})^{-1/2} \qquad (6)$$

where $t = 0$ and $\xi(0) = \xi_0$. ξ_0 is the proportion of $\vec{q}(0)$ that is contained in the full-blown psychopathological pattern $\vec{v}_k$, quantifying how much of the psychopathology is represented in the initial state of the person. The limit of (6) for large *t* becomes

$\xi(t) = \sqrt{\lambda/C}$, which is the full size of the order parameter ξ (7)

This means that the order parameter will always evolve to full size if there was any kind of overlap, even if very small, between an initial system state and the psychopathological pattern (Fig. 2). $\lambda t >> 1$ means that the larger the valence, the faster the attractor of the psychopathological pattern will be reached.

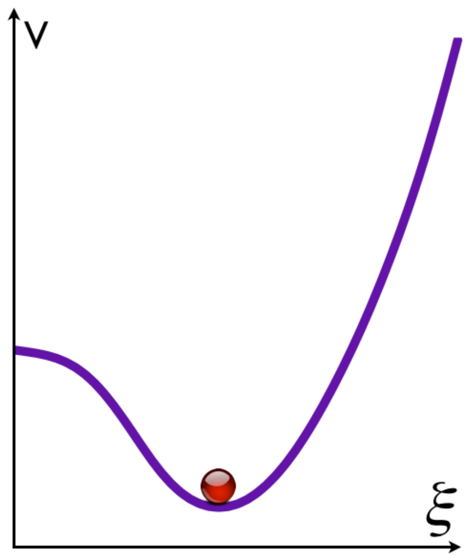

Figure 2. If only one pattern is established, and the initial order parameter is >0, the system (red ball) will always end up in the respective attractor, in the sink of potential V

Case 2: Two psychopathological attractors

We now discuss the case of two patterns (i.e. M=2). The psychopathology landscape of a person may accordingly contain two attractors, say one attractor of the original borderline personality disorder and a further attractor with a different, "healthy" constellation of attributes $\vec{q}$. Both patterns are described by their respective order parameters ξ_j (see Fig. 3).

$$\frac{d\xi_1}{dt} = \xi_1(\lambda_1 - B\xi_2^2 - C(\xi_1^2 + \xi_2^2)) \tag{8}$$

$$\frac{d\xi_2}{dt} = \xi_2(\lambda_2 - B\xi_1^2 - C(\xi_1^2 + \xi_2^2)) \tag{9}$$

The formal transformation of these equations yields

$$\xi_j(t) = \tilde{\xi}_j(t)\exp(-C\int_0^t(\xi_1^2(\tau) + \xi_2^2(\tau)d\tau)) \tag{10}$$

$$\frac{d\tilde{\xi}_1}{dt} = \tilde{\xi}_1(\lambda_1 - B\tilde{\xi}_2^2 \exp(-2C\int_0^t(\xi_1^2(\tau) + \xi_2^2(\tau))d\tau) \tag{11}$$

$$\frac{d\tilde{\xi}_2}{dt} = \tilde{\xi}_2(\lambda_2 - B\tilde{\xi}_1^2 \exp(-2C\int_0^t(\xi_1^2(\tau) + \xi_2^2(\tau))d\tau) \tag{12}$$

Equations (11) and (12) can be used to clarify the effects of competing order parameters $\xi_{1,2}$. In the following, we discuss several distinctions of Case 2 to observe the evolution of the system with respect to the relative size of the valences that are connected with the two order parameters.

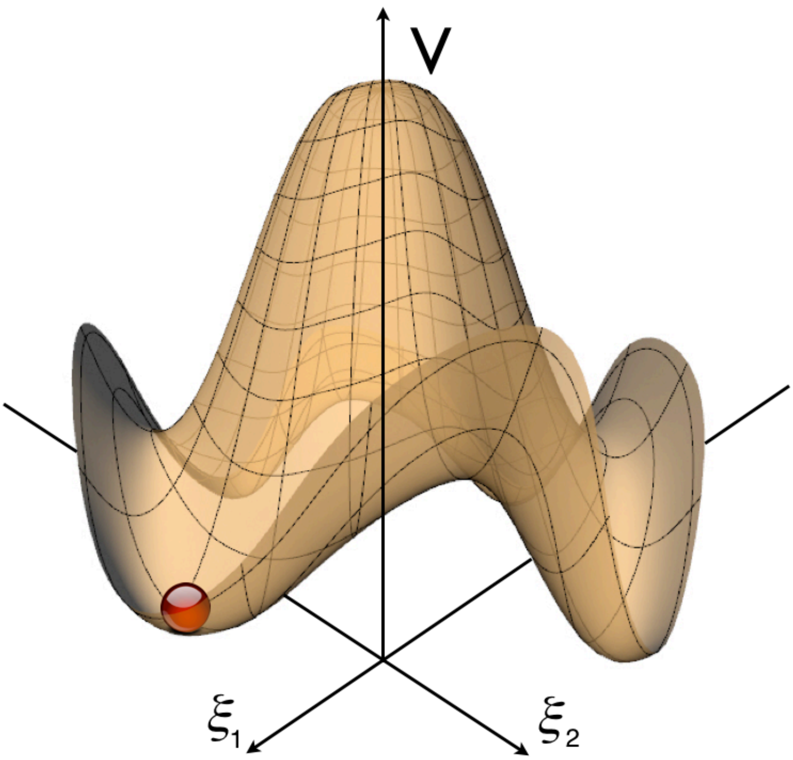

Figure 3. If two patterns are established, with their respective order parameters ξ_1, ξ_2, the system (red ball) will choose one of the attractors. V, potential

Case 2.1

In Case 2.1, the valence of pattern 1 is much larger than that of pattern 2: $\lambda_1 >> \lambda_2$. At the same time, however, the initial order parameters may be approximately equal:

$$\xi_1(0) \approx \xi_2(0) \text{ , i.e. } \tilde{\xi}_1(0) \approx \tilde{\xi}_2(0) \tag{13}$$

We solve the equations in small time-steps Δ. First step:

$$\frac{d\tilde{\xi}_1(0)}{dt} \approx \frac{\tilde{\xi}_1(\Delta) - \tilde{\xi}_1(0)}{\Delta} = \tilde{\xi}_1(0)(\lambda_1 - B\tilde{\xi}_2(0)^2) \tag{14}$$

or

$$\tilde{\xi}_1(\Delta) = \tilde{\xi}_1(0)(1 + \Delta(\lambda_1 - B\tilde{\xi}_2(0)^2) \tag{15a}$$

and

$$\tilde{\xi}_2(\Delta) = \tilde{\xi}_2(0)(1 + \Delta(\lambda_2 - B\tilde{\xi}_1(0)^2) \tag{15b}$$

Because of (13), the right-hand side of (15a) is larger than that of (15b), i.e. $\tilde{\xi}_1(\Delta) > \tilde{\xi}_2(\Delta)$. This argument holds for all time-steps when we replace B by

$$B(t) = B\exp(-2C\int_0^t \xi_1^2(\tau) + \xi_2^2(\tau)d\tau) \tag{16}$$

$$\tilde{\xi}_1(t) > \tilde{\xi}_2(t) \quad \text{and } \xi_1(t) > \xi_2(t) \tag{17}$$

for all times. At the end of the evolution, the system has settled in an attractor, and the change of order parameters becomes zero: $\frac{d\xi_j(0)}{dt} = 0$, $j = 1,2$ (18)

Thus

$$\xi_1(\lambda_1 - B\xi_2^2 - C(\xi_1^2 + \xi_2^2)) = 0 \tag{19}$$

$$\xi_2(\lambda_2 - B\xi_1^2 - C(\xi_1^2 + \xi_2^2)) = 0 \tag{20}$$

Can both patterns coexist, i.e. can both order parameters ξ_1 and ξ_2 be $\neq 0$?

$$(\lambda_1 - B\xi_2^2 - C(\xi_1^2 + \xi_2^2)) = 0 \tag{21}$$

$$(\lambda_2 - B\xi_1^2 - C(\xi_1^2 + \xi_2^2)) = 0 \tag{22}$$

The difference between (21) and (22) yields

$$\lambda_1 - \lambda_2 = B(\xi_2^2 - \xi_1^2). \tag{23}$$

This is in contradiction to (13). Therefore only one ξ_j can be $\neq 0$, and since $\xi_1 > \xi_2$ all the time, only ξ_1 wins. If $j = 1$ refers to the psychopathological pattern, it will win even if the order parameters were initially of the same size.

Case 2.2

We again discuss the case of two patterns (the first is the psychopathology disorder, the second a healthy attractor), which in the present case shall have valences equally large. The initial order parameter of the first pattern, however, is larger than that of pattern 2.

$$\lambda_1 = \lambda_2 = \lambda\,,\quad \xi_1 > \xi_2 \tag{24}$$

Again we start from (11), (12), which we solve in time-steps Δ in analogy to (15), (16). We discuss the right-hand side of

$$\tilde{\xi}_1(\Delta) = \tilde{\xi}_1(0)(1+\Delta(\lambda - B\tilde{\xi}_2(0)^2)) \tag{25}$$

$$\tilde{\xi}_2(\Delta) = \tilde{\xi}_2(0)(1+\Delta(\lambda - B\tilde{\xi}_1(0)^2)) \tag{26}$$

Because of (24), the factor of $\tilde{\xi}_1(0)$ in (25) is larger than that of $\tilde{\xi}_2(0)$ in (26), and furthermore $\xi_1(0) > \xi_2(0)$. Repeating this argument for all times and using (16), we obtain

$$\xi_1(t) > \xi_2(t) \text{ for all times} \tag{27}$$

In the final state we obtain

$$\xi_1(\lambda - B\xi_2^2 - C(\xi_1^2 + \xi_2^2) = 0 \tag{28}$$

$$\xi_2(\lambda - B\xi_1^2 - C(\xi_1^2 + \xi_2^2) = 0 \tag{29}$$

If both $\xi_1, \xi_2 \neq 0$ we obtain

$$B(\xi_2^2 - \xi_1^2) = 0 \tag{30}$$

in contrast to our finding (27). Thus again only ξ_1 survives. The result of Case 2.1 (Fig. 3) is hence repeated also when the valences for the healthy and the psychopathological attractor are equal, but the order parameter of the psychopathological pattern was initially larger.

Case 3: The general case of many patterns

In Case 3, the valence of pattern 1 is larger than either valence of the other patterns. There may be *M* patterns, $\lambda_1 > \lambda_k$, $k = 2,..,M$ (31)

We insert in equation (7)

$$\xi_k(t) = \tilde{\xi}_k(t)\exp(-(B+C)\int_0^t \sum_{k'} \xi_{k'}^2(\tau)d\tau) \tag{32}$$

and obtain

$$\frac{d\tilde{\xi}_k(t)}{dt} = \tilde{\xi}_k(t)(\lambda_k + B\sum_{k'} \xi_{k'}^2 - B\sum_{k'\neq k} \xi_{k'}^2) \tag{33}$$

or just

$$\frac{d\tilde{\xi}_k(t)}{dt} = \tilde{\xi}_k(t)(\lambda_k + B\xi_k^2) \tag{34}$$

In analogy to (19) we introduce

$$B(t) = B\exp(-(B+C)\int_0^t \sum_{k'} \xi_{k'}^2(\tau)d\tau) \tag{35}$$

Thus, (34) becomes

$$\frac{d\tilde{\xi}_k(t)}{dt} = \tilde{\xi}_k(t)(\lambda_k + B(t)\tilde{\xi}_k) \tag{36}$$

Now let us assume that initially all order parameters are of about equal size, that is

$$\xi_k(0) \approx \xi_0 \text{ for all } k \tag{37}$$

but that (31) holds. We compare the equations (36) for $k=1$ and $k \geq 2$.

$$\frac{d\tilde{\xi}_1(t)}{dt} = \tilde{\xi}_1(t)(\lambda_1 + B(t)\tilde{\xi}_1) \tag{38}$$

$$\frac{d\tilde{\xi}_k(t)}{dt} = \tilde{\xi}_k(t)(\lambda_k + B(t)\tilde{\xi}_k^2),\ k \geq 2 \tag{39}$$

The further analysis is precisely the same as in Case 2.1 above. We find that

$\tilde{\xi}_1(t) > \tilde{\xi}_k(t)$ and then

$$\xi_1(t) > \xi_k(t),\ k \neq 1 \text{ for all times.} \tag{40}$$

Let us look at the final state where $\frac{d\xi_k}{dt} = 0$ for all k (41)

From eq. (4) we obtain

$$\xi_k(t)(\lambda_k - B\sum_{k' \neq k}^{M} \xi_{k'}^2 - C\sum_{k'}^{M} \xi_{k'}^2) = 0 \tag{42}$$

How many patterns (order parameters) can coexist under the assumptions of Case 3? We compare (42) for $k=1$ with any other k. If $\xi_1 \neq 0$, $\xi_k \neq 0$, any other $k \neq 1$ then

$$(\lambda_1 - B\sum_{k' \neq 1} \xi_{k'}^2 - C\sum_{k'} \xi_{k'}^2) = 0 \tag{43}$$

and

$$(\lambda_k - B\sum_{k' \neq k} \xi_{k'}^2 - C\sum_{k'} \xi_{k'}^2) = 0 \tag{44}$$

Taking the difference between (43) and (44) yields

$$\lambda_1 - \lambda_k = B(\sum_{k'\neq 1} - \sum_{k'\neq k}) = B(\xi_k^2 - \xi_1^2) \tag{45}$$

which, due to (31) and (40), again implies a contradiction, so that because of (40) only ξ_1 survives. Since always $\xi_1 > \xi_k$, $k \neq 1$, we may exclude $\xi_1 = 0$ in all cases.

In other words, if an individual possesses, at some point in time, equally large tendencies (order parameters) towards a number of different attractors, the attractor connected with the largest valence will eventually win out nevertheless.

Results

We conducted a mathematical analysis of the temporal evolution of a system with one or more previously established patterns (attractors) and their corresponding valence parameters. The computation of various cases provided the following results: Case 1 showed that, if only one pattern with a valence parameter $\lambda_1 > 0$ exists, an arbitrarily small overlap between the previously learned pattern $\vec{v}_1$ and an initial state $\vec{q}$ suffices for this pattern to develop completely. This means, if $\vec{v}_1$ denotes a psychopathological pattern such as BPD, and no alternative pattern of non-problematic cognition-behavior-experience ($\vec{v}_2$) was pre-established in the individual, any minimal incident or situation that overlaps with problematic cognition-behavior-experience leads to the development of the full disorder. The disorder is hence chronic, and any initial state will be ensued by the eventual manifestation of the disorder.

Case 2.1 discussed two attractors, the one of the disorder $\vec{v}_1$ and one of an alternative prototype $\vec{v}_2$, whose order parameters are equally strong. Thus there are initially equal tendencies towards the disorder and the previously learned alternative cognition-behavior-experience pattern. If the valence of the disorder however is larger, the full disorder will nevertheless be restituted, and the alternative pattern will completely disappear (with its order parameter vanishing). Case 2.2 in turn shows this scenario is also true for equally large valences of the two patterns, but an initially larger problematic cognition-behavior-experience order parameter. This means, again, any incident or situation that overlaps with problematic cognition-behavior-experience entails the full disorder. Case 3 is a generalization of Case 2.1 for many different patterns $\vec{v}_k$, showing that in the presence of a number of further alternative (non-problematic) cognition-behavior-experience patterns and respective alternative valences $\lambda_k > 0$, but with λ_1 larger than each single other λ_k, even approximately equivalent situations will again result in a winning disorder $\vec{v}_1$.

Discussion

Let us discuss these findings in the light of therapeutic interventions. The general Case 3 showed that any therapeutic intervention with insufficient valence will not ward off the eventual emergence of $\vec{v}_1$ and accordingly the restitution of the full disorder. We consider this an important result. For successful corrective interventions, the creation of novel cognition-behavior-experience patterns whose λ_k-s are larger than the pathological valence λ_1 is necessary. Overcoming an established psychopathological pattern can therefore not be achieved by just offering a (possibly large) number of alternative competing patterns. From our model follows that it is preferable to intensively support a single alternative λ_k instead of a number of alternatives, as long as each of these has less motivational intensity than the disorder.

Therapeutic intervention may address both the alternative cognition-behavior-experience patterns (the order parameters) and the motivational valences of such patterns. Our analysis points out that both targets of intervention are necessary to counter an established disorder. Case 1 showed that if there is no alternative to the dysfunctional pattern, it will almost always reappear. Thus a therapy that only addresses the state vector $\vec{q}$ of the patient will likely not be successful – a behavior therapy restricting itself to influence a patient's symptomatic behavior and stimulus environment or a cognitive therapy restricting itself to influence a patient's cognition (and maybe experience) will not prevail. The systems-theoretical reason for this is that it is unrealistic to expect that the patient will never again encounter states that overlap with the disorder pattern. Yet any overlap results in relapse, as Case 1 showed.

Hence it is essential to create and train alternative patterns of cognition-behavior-experience and impart one (rather than several) of these alternative patterns with high valence. This is mirrored in the results of psychotherapy research that have shown the high importance of so-called 'common factors' of psychotherapy (Frank, 1971; Strauß, 2001), these being general ingredients such as a reliable therapeutic alliance (Tschacher, Haken, & Kyselo, 2015), instilment of hope in the patient, and affective engagement of the patient. Modern psychotherapies have integrated the common factors view, and obviously no longer claim that psychotherapy is just the learning of new behavior or the unlearning of 'false' cognitions. An integrated approach emphasizing the importance of changing the valence of an alternative pattern of cognition-behavior-experience is also supported by the present analysis on the basis of self-organization theory. Corrective intervention must be 'valent', hence work with a focus on affective experiencing, emotion regulation, and motivation.

Our mathematical model has favored a 'winner take all' scenario from the start because we believe that this is the most realistic connectionist model for mental/brain functioning in perception. Thus successful corrective interventions by psychotherapy must create novel states that can be 'winners'. This is consistent with the therapeutic principle of offering alternative behavioral, emotional, or cognitive options λ_k rather than trying to directly suppress λ_1, e.g. by punishment. Such reinforcement of an alternative pattern must keep in mind that it is more successful to reward (i.e. increase the valence of) a single alternative pattern than invest the same intensity of reinforcement in several or many alternative patterns. Expressing these thoughts in a more speculative fashion: First, successful therapeutic intervention is 'systemic' in that it must generate a new holistic pattern or attractor – rather than attempt to correct all the single features and symptoms of the disorder. In this sense, holistic common factors are more important than linear techniques with respect to the outcome of therapy (which is to overcome disorder). Second, successful therapeutic intervention means emotion regulation in the first place (Koole & Tschacher, in press), which arises in the context of a synchronizing therapeutic alliance; this again emphasizes the significance of common factors which can induce and increase the valence of the alternative pattern.

Our analysis may have limitations owing to the modeling assumptions that we made by using the SC algorithm. First, we made the assumption of the 'winner take all' order parameter competition. Yet we do not believe this is a serious limitation because biological evolution has not equipped living organisms with complicated attractor landscapes, where they risk ending up in the 'ghost states' of local minima because that would very likely be detrimental for the organism's environmental fit. Rather, the temporal generation of the state vector $\vec{q}$ is a result of the competition of order parameters, a very fast process. The stronger order parameter ξ, i.e. the one with the larger attached valence and/or the larger initial value $\xi(0)$, will immediately govern the system. This is generally true for complex systems close to critical instabilities, as modeled by synergetics. In this manner, decision times of organisms can be minimized, and decisions can be clear and unequivocal. In personality psychology, we may recognize this mode of operations in the unconscious and intuitive processing of the self (Kuhl & Beckmann, 1994). Therefore, we believe our seemingly strong assumption of 'winner take all' dynamics is well justified. A second point is maybe a more serious limitation of the present analysis – we were observing the evolution equations for the order parameters, where we treated the valences as constants. This second assumption may not hold in certain, especially in psychological, settings because the valences may themselves fade once the system has settled in an attractor. This is a frequent observation in Gestalt perception, e.g. of the Necker cube (Fig. 1): When the cube is viewed for a certain period of time, the λ_1 of one order parameter is gradually depleted, so that the second order parameter is realized subsequently. The result is an oscillation between the two different ways of viewing the Necker cube. Thus, we may extend the mathematical discussion to valences that are functions of time and/or of the order parameters, which will allow for the depletion of valences (the depletion of parameters in the context of brain dynamics and intentionality was discussed by Haken & Tschacher (2010) and Tschacher & Haken (2007); the interplay of order parameters and system parameters was recently modelled by Frank, 2015).

Thus, in conclusion, we may derive the following working hypotheses from the present mathematical analysis of a complex system with competing patterns: Mere avoidance of an established dysfunctional pattern is likely not successful. The linear mending and correcting of symptoms by exercising specific therapeutic techniques is likewise a deficient strategy if it is not complemented by common factors. The best therapeutic option is to generate and then focus on one alternative pattern, eventually furnishing it with higher valence than the valence of the problematic pattern. We suppose that in psychotherapy this is achieved by constructing a pattern specifically tailored to the individual patient, and this alternative pattern must then be equipped, in the context of the therapeutic alliance, with as much emotional and motivational valence as possible.

Acknowledgement

Artwork in Figure 3 created by Miro Bannwart (software, Rhinoceros 5.0). We thank two anonymous reviewers for helpful suggestions.

References

Choi, C.-H. (1997). Generalizations of the Lotka-Volterra population ecology model: Theory, simulation, and applications. *Nonlinear Dynamics, Psychology, and Life Sciences, 1*(4), 263-273.

Ditzinger, T., & Haken, H. (1989). Oscillations in the perception of ambiguous patterns – A model based on synergetics. *Biological Cybernetics, 61*, 279-287.

Frank, J. D. (1971). Therapeutic factors in psychotherapy. *American Journal of Psychotherapy, 25*, 350-361.

Frank, T. D. (2015). On the interplay between order parameter dynamics and system parameter dynamics in human perceptual-cognitive-behavioral systems. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*, 111-146.
Fuchs, A. & Haken, H. (1988). Pattern recognition and associative memory as dynamical processes in a synergetic system. *Biological Cybernetics, 60*, 17-22.
Gibson, J. J. (1979). *The Ecological Approach to Visual Perception*. Boston: Houghton Mifflin.
Gray, C. M., König, P., Engel, A. K., & Singer, W. (1989). Oscillatory responses in cat visual cortex exhibit inter-columnar synchronization which reflects global stimulus properties. *Nature, 338*, 334-337.
Guastello, S. J. (1995). *Chaos, Catastrophe, and Human Affairs: Applications of Nonlinear Dynamics to Work, Organizations, and Social Evolution*. Hillsdale: Erlbaum.
Haken, H. (1977). *Synergetics — An Introduction. Nonequilibrium Phase-Transitions and Self-Organization in Physics, Chemistry and Biology*. Berlin: Springer.
Haken, H. (2004). *Synergetic Computers and Cognition. A Top-Down Approach to Neural Nets.* Berlin: Springer
Haken, H. (2008). *Brain Dynamics. Synchronization and Activity Patterns in Pulse-coupled Neural Nets with Delays and Noise* (2nd ed.). Berlin: Springer.
Haken, H. & Tschacher, W. (2010). A theoretical model of intentionality with an application to neural dynamics. *Mind and Matter, 8*, 7-18.
Haken, H. & Tschacher, W. (2011). The transfer of principles of non-equilibrium physics to embodied cognition. In: Tschacher W & Bergomi C (eds.) *The implications of embodiment: Cognition and communication* (pp. 75-88). Exeter: Imprint Academic.
Kohonen, T. (1987). *Associative Memory and Self-Organization* (2nd Ed.). Berlin: Springer.
Koole, S.L. & Tschacher, W. (in press). Synchrony in psychotherapy: A new framework for understanding the therapeutic alliance. *Frontiers in Psychology, 7.*
Kuhl, J. & Beckmann, J. (eds.)(1994). *Volition and Personality*. Seattle: Hogrefe & Huber.
Lewin, K. (1936). *Principles of Topological Psychology*. New York: McGraw-Hill.
Strauß, B. (2001). Abschied vom Dodo-Vogel: Störungsspezifische versus allgemeine Therapie aus der Sicht der Psychotherapieforschung. *Psychotherapie, Psychosomatik, Medizinische Psychologie*, *51*, 425-429.
Tschacher, W. & Haken, H. (2007). Intentionality in non-equilibrium systems? The functional aspects of self-organized pattern formation. *New Ideas in Psychology, 25*, 1-15.
Tschacher, W., Haken, H., & Kyselo, M. (2015). Alliance: A common factor of psychotherapy modeled by structural theory. *Frontiers in Psychology*, *6*, 421.